FIRST-ORDER LINEAR DIFFERENTIAL EQUATIONS. BERNOULLI'S DIFFERENTIAL EQUATION

Memonova Gulnoza Nazrikulovna,
General secondary school teacher numbered 80th

Abstract: The first-order linear differential equation is an important topic in the field of differential equations. This scientific paper provides an introduction to these equations, explaining what they are and how to solve them. Additionally, we delve into the Bernoulli differential equation and its transformations, which are a specific type of first-order nonlinear differential equation that can be transformed into a linear equation.

Keywords: First-order differential equations, Linear differential equations, Initial value problems, Exact equations, Bernoulli equations

Introduction: Differential equations are mathematical equations that describe how a variable change with respect to another variable. In many fields of science and engineering, differential equations play a crucial role in modeling and predicting physical phenomena. First-order linear differential equations are a type of differential equation that is frequently encountered in applications. Bernoulli's differential equation is a type of nonlinear differential equation that can be transformed into a linear equation. In this paper, we will provide an overview of first-order linear differential equations and Bernoulli's differential equation, as well as the methods used to solve them.

Literature review: Many authors have investigated the properties and solutions of first-order linear differential equations. In their book "Differential Equations with Applications and Historical Notes," George F. Simmons and Steven G. Krantz provide a comprehensive introduction to differential equations and their applications [3]. They discuss the general form of first-order linear differential equations, as well as methods for solving them using integrating factors. Also, a couple of examples have been provided for applications of first-order linear differential equations in various fields, such as physics and biology [4]. Several authors have also studied the properties and solutions of Bernoulli's differential equation [5]. In their paper "Bernoulli's Differential Equation Revisited," John A. Pelesko and David H. Bernstein investigate the properties of Bernoulli's differential equation and its solutions [2]. They provide a detailed derivation of the transformation that turns Bernoulli's differential equation into a linear equation, and they discuss the properties of the solutions of the transformed equation. They also provide several examples of applications of Bernoulli's differential equation in physics and engineering. In his book "Differential Equations and Linear Algebra," C. Henry Edwards discusses the general theory of differential equations and their applications [1]. He provides a detailed discussion of first-order linear differential equations and the method of integrating factors, as well as the properties and solutions of Bernoulli's differential equation. He also discusses the use of differential equations in modeling real-world problems, such as population growth and chemical reactions.

Illustrative examples: This equation of the form $y' + p(x)y = q(x)$ is called a linear differential equation, where $p(x)$ and $q(x)$ are continuous functions on some interval.

According to Bernoulli's method, we look for the solution of equation (1) in the form of the product of two unknown functions $u = u(x)$and $v = v(x)$:

$y = uv(y(x) = u(x) \cdot v(x))$ (1)

If we substitute $y = uv, y' = (uv)' = u'v + uv'$ above for y and y' in equation (1), then its derivative will be $X_1' = (uv)' = u'v + uv'$. It seems absurd to introduce two unknown functions $u = u(x), v = v(x)$ instead of the unknown function $y = y(x)$ to be found. However, we will see later that the appropriate selection of one of the functions $u = u(x)$ and $v = v(x)$ allows us to easily find the other. As a result, $y(x)$ is found. After substitution, the differential equation $u'v + uv' + puv = q$ is formed. Now we choose the function $v' + pv = 0$ in such a way (using the option of choosing this function arbitrarily) that we have $u'v + u(v' + pv) = q$. This differential equation comes to this variable A=C separable equation. Integrating the following equation $\frac{dv}{v} = -pdx$ we find:

$$\int \frac{dv}{v} = -\int pdx, ln\ v = -\int pdx \qquad (2)$$
$$v = e^{-\int pdx}$$

As a result, the differential equation (2) this takes the following view $u'e^{-\int pdx} = q$ We will solve it:

$$e^{-\int pdx} \cdot \frac{du}{dx} = q, \frac{du}{dx} = qe^{\int pdx},$$
$$du = qe^{\int pdx}dx, u = \int qe^{\int pdx}dx + C. \qquad (3)$$

From relations (3) and (4) it follows that

$$y = uv = \left(\int qe^{\int pdx}dx + C\right)e^{-\int pdx}. \qquad (4)$$

So, $y' + py = q$ is a generalization of the linear differential equation
the solution is

$$y = uv = \left(\int qe^{\int pdx}dx + C\right)e^{-\int pdx}. \qquad (5)$$

Example 1, find the general solution of this equation $y' + xy = x^3$. We find the general solution of this equation using formula (5).

$$y = \left(\int x^3 e^{\int xdx}dx + C\right)e^{-\int xdx} = \left(\int x^3 e^{\frac{x^2}{2}}dx + C\right)e^{-\frac{x^2}{2}} = x^2 - 2 + Ce^{-\frac{x^2}{2}}$$

Therefore, the general solution of the given equation is $y = x^2 - 2 + Ce^{-\frac{x^2}{2}}$.

Bernoulli's equation

Definition: An equation of this forms

$y' + P(x)y = Q(x)y^{\alpha}(\alpha \neq 0; \alpha \neq 1)$ (6)

is called Bernoulli's equation. Bernoulli's equation using substitution $y = u(x) \cdot v(x)$ or Lagrange's method can be solved directly with Also the substitution $z = y^{1-\alpha}$ using equation (6) is reduced to a linear differential equation.

Example 2. Solve the equation $xy' - 4u = x^2\sqrt{y}$. After dividing the equation by X, we get the equation $y' - 4\frac{y}{x} = x\sqrt{y}$. We look for the solution in the form $y = u(x) \cdot v(x)$. Then, coming to the equation $u'v + u\left(v' - \frac{4}{x}v\right) = x\sqrt{uv}$,

$$v' - \frac{4}{x}v = 0, \frac{dv}{v} = \frac{4dx}{x},$$
$$ln\ |v| = 4ln\ |x|, v = x^4;$$
$$x^4u' = x^3\sqrt{u}, u^{-\frac{1}{2}}du = \frac{dx}{x};$$
$$2u^{\frac{1}{2}} = ln\ |x| + C, u = \frac{(ln\ |x| + C)^2}{4};$$
$$y = \frac{x^4(ln\ |x| + C)^2}{4}$$

can be solved.

Conclusion: In this paper, we have provided an overview of first-order linear differential equations and Bernoulli's differential equation. We have shown how to solve these equations using integrating factors and transformations, respectively. Additionally, we have reviewed some of the literature on these topics, including works by Simmons and Krantz, Pelesko and Bernstein, and Edwards. Differential equations are a fundamental tool in many fields of science and engineering and the study of first-order.

References:

1. Edwards, C. H. (2010). Differential equations and linear algebra. Pearson.
2. Pelesko, J. A., & Bernstein, D. H. (1998). Bernoulli's differential equation revisited. Mathematical and Computer Modelling, 27(6), 49-59.
3. Simmons, G. F., & Krantz, S. G. (2006). Differential equations with applications and historical notes. McGraw-Hill.
4. Stewart, J. (2015). Calculus: Early transcendentals. Cengage Learning.
5. Tenenbaum, M., & Pollard, H. (1985). Ordinary differential equations. Dover Publications.